THE IMPORTANCE OF SKIN-EFFECT IN MICROSTRIP LINES AT HIGH FREQUENCIES

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ABSTRACT

The higher speeds and the larger device densities in modern integrated circuits demand better characterization of the electrical parameters which can influence their performance. In this paper, the importance of the skin-effect is examined, and an integral equation approach is introduced for the calculation of the ac resistance and reactance.

INTRODUCTION

During the last few years, semiconductor technology is pushing towards higher speeds and larger device densities. As a result, we are moving towards pulses with risetimes faster than one nanosecond, that is, signals with significant power up to ~2 GHz. It is then apparent that, with metallization thicknesses below 10 μm, the skin depth becomes significant even at the upper end of the pulse spectrum. Up to now, the loss calculations for high frequencies have been primarily based on Wheeler's incremental inductance rule [1]. The requirement for Wheeler's theory to be applicable is that the cross-sectional dimensions of the conductor are large compared to the skin depth (~4 times the skin depth). This theory no longer holds when the skin depth is in the order of the metallization thickness, which is the case of modern integrated circuits, especially at lower frequencies (500 MHz - 1 GHz). In this paper, an integral equation formulation of the skin-effect problem is presented. The objective is to calculate the current distribution for any arbitrary combination of conducting strips. Once these distributions are found, the ac resistances and inner reactances can be computed, and can be used as input parameters to CAD tools for the analysis of multiple coupled transmission lines [2].

MATHEMATICAL FORMULATION

Let us consider the microstrip configuration of Fig. 1. The conductors are parallel to the z-axis and have magnetic permeability μ_0, an electric conductivity σ_n, and a cross section S_n

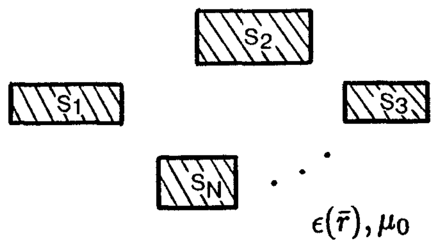

Fig. 1

($n = 1, 2, \ldots, N$). The conductors are carrying the alternating currents $i_n(t) = I_n \cos(\omega t + \phi_n)$ ($n = 1, 2, \ldots, N$), and are surrounded by an inhomogeneous dielectric with magnetic permeability μ_0. For the special case of the two-dimensional configuration, the current density has only one component in the z direction, and the electric field inside the conducting strip is given by

$$E_z(x,y) = -j\omega A_z(x,y) - \frac{dV}{dz} \qquad (1)$$

where $A_z(x,y)$ is the magnetic vector potential and $V(x,y,z)$ is the electric potential. At any point inside the conductor dV/dz is a complex constant related to the specific choice of the reference for the magnetic vector potential [3]. From (1) using Ohm's law we can derive the equation for the current density for each point inside any of the conductors,

$$J_n = -j\omega\sigma_n A - \sigma_n \frac{dV}{dz}, \qquad n = 1, 2, \ldots, N \qquad (2)$$

where the subscript z has been dropped for simplicity. If an average value of the magnetic vector potential is defined over the cross-section of each conductor by

$$\tilde{A}_n = \frac{1}{S_n}\int_{S_n} A(x,y)dxdy, \quad n = 1,2,\ldots,N \quad (3)$$

then the current density inside any of the conductors is given by

$$J_n = -j\omega\sigma_n A + j\omega\sigma_n \tilde{A}_n + \tilde{J}_n, \quad (4)$$

where $\tilde{J}_n$ is the average current density distribution in the n^{th} conductor, that is, $\tilde{J}_n = I_n/S_n$, $n = 1,2,\ldots,N$ [4].

On the other hand, the magnetic potential at any point (x,y) is given by

$$A(x,y) = -\frac{\mu_0}{2\pi}\int_S J(x',y')\ln R dx'dy', \quad S = \sum_{n=1}^{N} S_n \quad (5)$$

where $R = \sqrt{(x-x')^2 + (y-y')^2}$. It is now apparent that substitution of (3),(5) in (4) results in an integral equation for the current densities inside the conductors.

The integral equation is solved numerically using the method of moments. Of major importance from a computational point of view is the size of the matrix which results from the application of the method of moments. Since the dimension of the matrix is proportional to the number of subintervals used in the discretization of the cross sections of the conductors, it is apparent that extra caution is needed in the way that this discretization is performed in order to keep the size of this matrix as small as possible, while preserving the accuracy of the numerical solution. For example, as the frequency of interest increases, most of the conductor current concentrates close to the surface and very little flows inside the conductor. This then suggests a nonuniform discretization of the cross section with a mesh that is denser close to the surface of the conductor.

NUMERICAL RESULTS

Once the current distributions have been found, the computation of the ac resistance of the n^{th} conductor is straightforward,

$$R_{ac}^n = \frac{1}{\sigma_n}\frac{\int_{S_n}|J_n|^2 ds}{\left(\int_{S_n} J_n ds\right)^2} \quad (6)$$

Our preliminary results are for the case of a single strip of rectangular cross section. In Fig.2 the computed ac resistance normalized to the dc resistance is shown as a function of the normalized frequency $f_p = \sqrt{2f\sigma\mu_0 S}$ with the shape factor a/b as parameter. The solid lines show the measured alternating current resistances published by Haefner [6]. We see that the agreement with our numerical results is excellent, and this confirms our theoretical formulation.

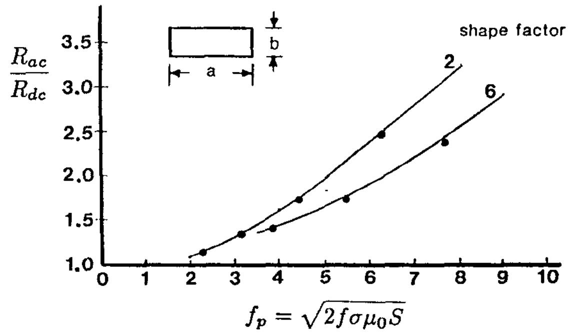

Fig.2

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